(PMAT 201)

SRR & CVR Government Degree College (A): Vijayawada

November 2021-BATCH 2020-21

Department of Mathematics

COMPLEX ANALYSIS

Semester -II: End Examination

Time: 3 Hours **Max. Marks: 60**

I. Answer any 5 questions out of the 10 short answer questions (5x4=20 Marks)

1. Write the polar form of C-R equations.
2. Show that $u(x,y) = x^3 - 3xy^2$ is harmonic function. Find an analytic function whose real part is $u(x,y)$.
3. State and prove Liouville's theorem
4. Evaluate $\int_C \frac{z+4}{z^2+2z+5} dz$ where C is the circle |z|=1.
5. Expand the function $f(z) = \frac{1}{z^2(1-z)}$
6. Determine the Singular points of $f(z) = \frac{z^2+1}{z^2+2z+2}$
7. Define singularity of a function. Classify the singularities of (i)$sin\frac{1}{z}$ and (ii) $log\frac{z+1}{z^2}$
8. Discuss the transformation $w = z^2$
9. Define Mobius transformation. Find the mobius transformation which maps the points 1,i,-1 to i,-1,1.
10. Show that the mapping $w = \frac{1}{z}$ transforms circles into circles.

II. Answer any 5 questions out of the 10 internal choice questions (5x8=40 Marks)

UNIT-I

11. If f is an analytic function, with usual notation prove that $\left(\frac{d^2}{dx^2} + \frac{d^2}{dy^2}\right)|f(z)|^2 = 4.|f'(z)|^2$

(OR)

12. State and prove chain rule in analytic functions.

(PTO)

UNIT-II

13. State and prove Cauchy Goursat theorem.

(OR)

14. Let C be a straight line segment from 2 to 2+i then establish that $\left|\int_C \frac{1}{z^2+1}dz\right| \leq \frac{1}{2\sqrt{5}}$.

UNIT-III

15. Find the Laurents series expansion of $f(z) = \frac{1}{z(z-1)(z-2)}$ in (i) ann(0,0,1) and (ii) ann(0,1,2).

(OR)

16. State and Prove Taylor's theorem.

UNIT-IV

17. Evaluate the integral $\int_C \frac{5z-2}{z(z-1)}dz$ where C is the circle $|z| = 2$, described counter clockwise.

(OR)

18. Evaluate $\int_0^{2\pi} \frac{d\theta}{1-2p\cos\theta+p^2}$, $0 < p < 1$ using the theory of residues.

UNIT-V

19. State and prove augment principle.

(OR)

20. State Rouche's theorem. Show that the equation $e^z = az^n$, has n roots inside the unit circle if $a > e$.

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(PMAT 202)

SRR & CVR Government Degree College (Autonomous): Vijayawada

November 2021-BATCH 2020-21

Department of Mathematics

NUMERICAL METHODS

Semester -II: End Examination

Time: 3 Hours **Max. Marks: 60**

I. Answer any 5 questions out of the 10 short answer questions **5 X 4 = 20Marls**

1. Using iteration method find $x^3 + x^2 - 1$.
2. Define Rate of convergence.
3. Describe the principle involved in Guass Jordan Method for finding the inverse of a matrix.
4. State Guass -Jacobi formula for solving a system of three algebraic equations.
5. State Newton's forward difference interpolation formula to compute the first order derivative of $y = f(x)\ at\ x = x_0$.
6. Use Taylor's series method to find y(0.1) given $\frac{dy}{dx} = x^2 + y^2, y(0) = 1$.
7. State the finite difference method for solving wave equation.
8. Using the operator relations, derive approximations to the derivatives $f'(x_n) \& f''(x_n)$ in terms of the backward differences.
9. Using Simpson's 1\3 rule find $\int_0^1 \frac{1}{1+x} dx$ by taking four equal sub intervals.
10. Evaluate $\int_0^6 \frac{1}{1+x^2} dx$ by trapezoidal rule with h=1.

II. Answer any 5 questions out of the 10 internal choice questions **5x8=40 Marks**

UNIT-I

11.a) Find the root of the equation $x^3 - 2x - 5 = 0$ using false-position correct to two decimal places.

(OR)

(P.T.O)

b). Use Newton-Raphson method to obtain a root, correct to three decimals of the equation $x + logx = 2$.

UNIT-II

12.a). Find the eigenvalues and eigenvectors of the matrix $A = \begin{matrix} 1 & \sqrt{2} & 2 \\ \sqrt{2} & 3 & \sqrt{2} \\ 2 & \sqrt{2} & 1 \end{matrix}$ by using Jacobi method.

(OR)

b). Solve the equations $27x + 6y - z = 85,\ 6x + 15y + 2z = 72, x + y + 54z = 110$ by using Guass Siedal Method.

UNIT-III

13.a). For the following data, Calculate the differences and obtain the forward and backward difference polynomials, interpolate at x=0.25 and x=0.35.

x	0.1	0.2	0.3	0.4	0.5
F(x)	1.40	1.56	1.76	2	2.28

(OR)

b). State and prove Newton's divided difference formula.

UNIT-IV

14.a).Given the following values of $f(x) = \log_e x$ find the approximate values of $f'(2.0)$ using linear and quadratic interpolation and $f''(2.0)$ using quadratic interpolation.

i	0	1	2
x_i	2.0	2.2	2.6
$f(x_i)$	0.69315	0.7885	0.9555

(OR)

(P.T.O)

b). Find $f'(0.6)$ and $f''(0.6)$ from the following data by using Stirling's formula

x	0.4	0.5	0.6	0.7	0.8
F(x)	1.5836	1.7974	2.0442	2.3275	2.6510

UNIT-V

15.a). Using Euler's modified method, solve the equation $y' = x + |\sqrt{y}|$ with y(0) =1 for $0 \le x \le 0.2$.

(OR)

b). Using Runge Kutta method find the solution of the differential equation $y' = 3x + \frac{1}{2}y$ with $y_0 = 1$ at x = 0.1.

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(PMAT 203)

SRR & CVR Government Degree College (Autonomous): Vijayawada

November 2021-BATCH 2020-21

Department of Mathematics

PARTIAL DIFFERENTIAL EQUATIONS

Semester -II: End Examination

Time: 3 Hours **Max. Marks: 60**

PART-A

I. Answer Any 5 Questions Out Of The 10 Short Answer Questions **5×4=20marks**

1. Solve $\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)}$

2. Determine whether the following equation is integrable or not

$$(y^2+xz)\,dx + (x^2+yz)\,dy + 3z^2dz = 0$$

3. Find the general integral of the Linear Partial differential equations: $y^2p - xyq = x(z-2y)$

4. Find the complete integral of the equation

$$(p+d)(z-xp-yq) = 1$$

5. Verify that the Partial Differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = \frac{2z}{x^2}$ Is satisfied by $z=\frac{1}{x}\emptyset(y-x) + \emptyset^1(y-x)$, where $\emptyset$ is an arbitrary function.

6. Classify the second order equations of the type $Rr+Ss+Tt+f(x,y,z,p,q)=0$ by their canonical forms

7. Write the laplace's equation in
a. Spherical polar coordinates $(r,\theta,\emptyset)$
b. Cylindrical Coordinates$(\rho, \emptyset, z)$

8. State Kelvin's inversion Theorem.

9. What is Helmholtz's equation ? Give twoconcurrences of the wave equation in Physics ?

10. Show that $y = A(p)e^{ip(t\pm x/c)}$ in a Solution of the wave equation for arbitary forms of the function A Which depends only on A.

(P.T.O)

PART-B

II. Answer any 5 questions out of the 10 internal choice essay question **5X8=40marks**

11. a) Find the orthogonal Trajectories on the sphere $x^2+y^2+z^2=a^2$ of its intersection with the paraboloids $Xy=cz$, c being a parameter

(or)

b). Solve the equation
$Yz(y+z)dx + xz(x+z)dy + xy(x+y)dz=0$ By verifying that the given equation is integrable.

12 a). Find the integral surface of the linear partial Differential equation
$X(y^2+z)p - y(x^2+z)q = (x^2-y^2)z$ which Contains the straight line $x+y=0$, $z=1$

b). Find the Complete integral of the equation $(p^2+q^2)y=qz$ by charpit's method

13 a) 1. Find the particular integral of the equation

$$(D^2-D^1)^2 = 2Y - X^2$$

(or)

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} = x-y$$

2. Find the complementary function of the equations

(or)

b). Reduce the equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = 0$ to canonical form and hence solve it.

14 a). Show that the surfaces $(x^2+y^2)^2-2a^2(x^2-y^2)+a^4=0$ can form a fanily of equipotential surfaces and find the general form of the corresponding potential function.

(or)

b). Obtain the solution of Laplace's equation in spherical Polar Coordinates.

15 a). Derive the solution of the one – dimensional wave equation.

(or)

b). Obtain Riemann-Volterra's solution for one – dimensional wave equation.

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(PMAT 204)

SRR & CVR Government Degree College (Autonomous): Vijayawada

November 2021-BATCH 2020-21

Department of Mathematics

LATTICE THEORY

Semester -II: End Examination

Time: 3 Hours Max. Marks: 60

PART - A

I. Answer any 5 questions out of the 10 short answer questions 5 X 4 = 20Marls

1. Draw the hasse diagram for the ordered set of all divisors of 60.
2. For any elements a,b of a lattice $a \cap b = b \Leftrightarrow b \cup a = a$.
3. Prove that every weakly complemented lattice is semi complemented.
4. Draw the hasse diagram of the lattice $\{P(S), \subseteq\}$ where $S = \{a, b, c\}$ and P(S) is the power set of S.
5. Describe the sets by regular expressions (i) $\{abba\}$ & (ii) $\{01,10\}$.
6. Show that the join of two compact elements of a lattice iself is compact.
7. Prove that every sublattice and every homomorphic image of a distributive lattice is distributive.
8. When is lattice said to be bounded and distributive.
9. Show that every distributive lattice is modular.
10. Show that every Boolean ring is commutative and is of characteristic of 2.

PART - B

II. Answer any 5 questions out of the 10 internal choice questions 5x8=40 Marks

UNIT-I

11.a) State the chain axiom and derive Kuratowski-Zorn from the chain axiom.

(OR)

(P.T.O)

b) If a partially ordered set P satisfies the minimum condition then prove that to each $x \in P$, there exists atleast one minimal element $m \in P$ such that $x \geq m$.

UNIT-II

12 a) State the principle of duality with respect to Lattices.

(OR)

b) Prove that two lattices are isomorphic if and only if they are also order isomorphic.

UNIT-III

13 a) Show that the every lattice satisfying either the maximum or the minimum condition is conditionally complete.

(OR)

b) Give an example of endomorphism of a complete lattice which is idempotent.

UNIT-IV

14 a) Prove that a lattice is modular if and only if every triplet of elements a, b, c $(a \leq c)$ has a median.

(OR)

b) State and prove Dedikind's modularity criterian.

UNIT-V

15 a) In every Boolean algebra, verify that

(a) $med(a', b', c') = (med(a, b, c))'$

(b) $(x \wedge y') \vee (x' \wedge y) = (x \wedge y) \wedge (x \vee y') \vee (x \wedge y)' \wedge (x \wedge y)$

(c) $x \leq y \Leftrightarrow x \wedge y' = 0 \Leftrightarrow x' \vee y = i$.

(OR)

b) Define valuation of a Boolean algebra. Prove that a valuation v of a Boolean algebra is additive if and only if v (0) = 0.

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(POMAT 207)

SRR & CVR Government Degree College (Autonomous): Vijayawada

November 2021-BATCH 2020-21

Department of Mathematics

GRAPH THEORY

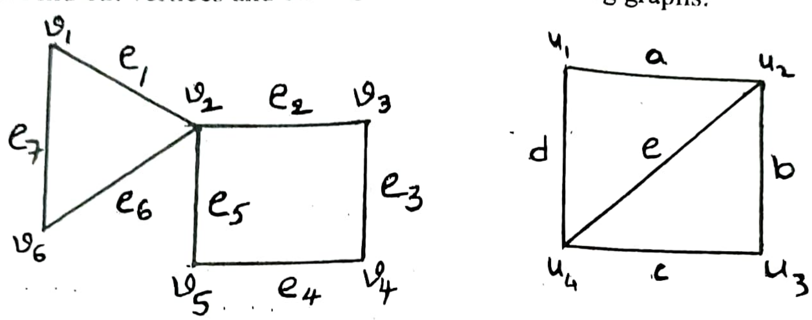
Semester -II: End Examination

Time: 3 Hours **Max. Marks: 60**

PART - A

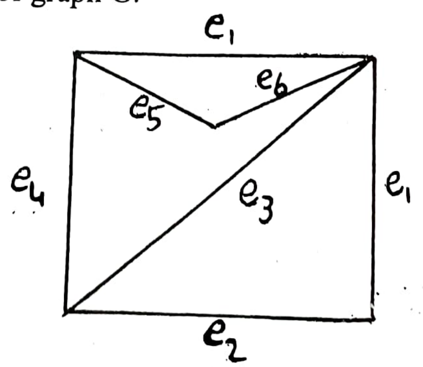
I. Answer Any 5 Questions Out Of The 10 Short Answer Questions **5×4=20marks**

1. Explain Konigsberg Bridge Problem?
2. Define Euler Graph and draw an example for it.
3. Prove that there is one and only one path between every pair of vertices in a tree T.
4. Find cut vertices and cut edges from the following graphs.



5. What is a planner graph ? Draw examples for planner and Non-planar graphs.

6. Draw dual of graph G.



7. Define circuit matrix and path matrix.
8. What is a directed graph ?
9. Define chromatic number ?
10. State Max – flow Min-cut theorem.

(P.T.O)

PART B

II. Answer any 5 questions out of the 10 internal choice essay question **5X8=40marks**

11 a) i. Prove that $\sum_{i=1}^{n} d(\theta i) = 2e$

ii. Prove that the number of vertices of odd degree in a graph is always even.

(or)

b). Prove that a given connected graph G is an EULER GRAPH if and only if all vertices of G are of even degree.

12. a) i. Prove that there are atleast two pendent vertices in any tree.

ii. Prove that every tree has either one or two centers.

(or)

b). Prove that the vertex connectivity of any graph G can never exceed the edge connectivity of G.

13 a). Prove that a connected planar graph with n vertices and e – edges has e -n+2 regions.

b). Prove that a graph has a dual if and only if it is planar.

14 a). Of B is a circuit matrix of a connected graph G with **e** edges and **n** vertices. Prove that rank of B=e-n+1.

(or)

b). Prove that the determinant of every square sub-matrix of A, the incidence matrix Of **a** digraph is 1,-1 or 0 .

15 a) i. Prove that every tree with two or max vertices is 2 – chromatic.

ii. Explain four-color problem.

(or)

b). Prove that in a given transport network G, the value of flow W From source S to sink **t** is less than or equal to the capacity of any curt separating **s** from t.

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